PULSE-COUPLED CLOCKS
OVERVIEW

Introduction

Pulse-Coupling Synchronization

Pulse-Coupled Integrate-and-Fire Oscillators

Pulse-Coupled Distributed Discrete PLLs

Impact of Topology and Shadowing
Synchronization: the process of achieving and maintaining coordination among independent local clocks via the exchange of local time information

Many applications require some common time scale between the nodes

New emerging challenges: energy efficiency, scalability, and application specificity
• Different approaches are used to mutual time synchronization, but the two main families of techniques

- **Packet-Coupling**: consists in the periodic exchange of packets carrying timestamps that contain the local time $t_j(n)$ at the sender

- **Pulse-Coupling**: each node sends a train of waveforms $g(t)$ for every tick of the local clock
PULSE-COUPLED SYNCHRONIZATION

- It is a physical layer-based scheme
- Local timing information is encoded directly in the transmission times of given waveforms → each node transmits a periodic train of waveforms $\sum n g(t - t_i(n))$ according to its own local clock
- The local clock of every node is updated after the nodes receive the signal
- It is scalable and have limited complexity

$t_i(n)$ - the time of the $n$-th tick of the $i$-th clock ($i = 0, 1, ..., N$)

$N$ - the total number of nodes
The two main implementations of the pulse coupled clock synchronization strategy are:

- Integrate-and-fire oscillators
- Distributed Discrete PLL

Assumptions:

- the absence of phase noise and delays
- each node is able to detect the time of arrival $t_i(n)$ of any pulse received
- all the nodes have the same frequency ($T_i = T_{nom}$)
PULSE-COUPLED INTEGRATE-AND-FIRE OSCILLATORS

- Each node has an integrate-and-fire oscillator: an integrator block and a trigger
- It is described by a state variable \( x_i(t) = g(\Phi_i(t)) \)
  - \( g(\cdot) \) - a periodic function, monotonically increasing from 0 to 1, and concave in a period of \( 2\pi \)
  - \( \Phi_i(t) \) - a phase
PULSE-COUPLLED INTEGRATE-AND-FIRE OSCILLATORS

- Upon detection of the pulse sent by any node $j$ at time $t_j(n)$, the $i$-th clock modifies the state function by adding a correction or adjustment factor $\varepsilon$ towards the goal of selecting a firing instant that is closer to that of clock $j$. In this way the phase $\Phi_i(t)$ is adjusted.

- The convergence and the stability can be studied using Lyapunov stability.
The main drawbacks of this model when applied to wireless networks are:

1) hard to extend the analysis to realistic and complex scenarios with:
   - inaccurate clocks,
   - propagation delays, or
   - time-varying channels;

2) the system design is not flexible enough to grant degrees of freedom for the achievement of additional relevant goals (trading complexity for accuracy, security...)
Nodes exchange the local time information with its neighboring nodes.

Each node calculates the time difference (time difference detector - TD) based on the received signal.

The result is fed to a loop filter $\varepsilon(z)$.

Eventually, the output of the loop filter is fed to the VCC that updates the clock.
PULSE-COUPLED DISTRIBUTED DISCRETE PLL LOOP FILTER

1. Simple filter $\varepsilon(z) = \varepsilon_0$
   → the stability under the same conditions as the previous method.

2. Moreover, a pole can be added in the loop filter $\varepsilon(z) = \varepsilon_0 \frac{z^{-1}}{1-\mu z^{-1}}$
   → reduce the offset in steady-state conditions losing some phase margin.
PULSE-COUPLED DISTRIBUTED DISCRETE PLL LOOP FILTER

3. Further improvements can be achieved using a PI regulator as a loop filter $\epsilon(z) = \epsilon_0 \frac{1-z^{-1}}{1-\mu z^{-1}}$

$\rightarrow$ cancel the offset and obtain the full synchronization between nodes

THE LOCAL NOMINAL FREQUENCY EQUALS THE COMMON ONE AND THERE IS NO MISMATCH BETWEEN THE TICKS OF THE CLOCK
IMPACT OF TOPOLOGY AND SHADOWING

• The convergence properties of distributed synchronization depend on the topology of the network of randomly located nodes with weights $\alpha_{ij}$

• Assuming log-normal shadowing and considering the clock standard deviation $\xi(n)$ of a pulse-coupled PLLs → the power received over distance $d_{ij}$ is:

$$P = 10^{\frac{\nu}{10}}/d_{ij}$$

where $\nu$ a zero-mean Gaussian random variable with standard deviation $\sigma$
• The shadowing in the model ($\sigma$) improves:
  → convergence speed
  → phase error

• Small-world network: where the paths are made of a small number of edges between any two nodes

• Shadowing enhances the small-world properties of the connectivity graph
REFERENCES
