PULSE-COUPLING SYNCHRONIZATION

Pulse-coupling is a physical layer-based scheme, where the local timing information is encoded directly in the transmission times of given waveforms. It is scalable since the operations performed at each node are independent of the number of nodes available in the network and have limited complexity, requiring only simple processing at the baseband level.

Each node transmits a periodic train of waveforms \( \sum_{n} g(t - t_i(n)) \) according to its own local clock, as shown in Figure 1, where \( t_i(n) \) as the time of the \( n \)-th tick of the \( i \)-th clock (\( i = 0, 1, \ldots, N \), where \( N \) is the total number of nodes), and \( T_i \) is the local time period for each node \( (T_i = t_i(n) - t_i(n - 1)) \). Figure 1 depicts the three nodes \( (N = 3) \) where each node transmits the signal \( g(t) \) for every tick of their local clock.

The local clock of every node is updated after the nodes receive the signal, which is a combination of waveforms transmitted by adjacent nodes. The update is performed by processing the received signal.

The two main implementations of the pulse-coupled clock synchronization strategy are Integrate-and-fire oscillators and Distributed Discrete PLL.

Some assumptions are made, such as the absence of phase noise and delays. It is also assumed that each node is able to detect the time of arrival \( t_j(n) \) of any pulse received from its adjacent nodes with infinite time resolution, and that all the nodes have the same frequency \( (T_i = \frac{T_{nom}}{}). \)
PULSE-COUPLED INTEGRATE-AND-FIRE OSCILLATORS

Each node has an integrate-and-fire oscillator, composed by an integrator block and a trigger, that can be described by a state variable \( x_i(t) = g(\Phi_i(t)) \), where \( g(\cdot) \) is a periodic function (with period \( 2\pi \)) such that in each period it is monotonically increasing from zero to one, and concave, as shown in figure 2(a).

Figure 2(b) shows the working principle of the coupling mechanism: upon detection of the pulse sent by any node \( j \) at time \( t_j(n) \), the \( i \)-th clock modifies the state function by adding a correction or adjustment factor \( \varepsilon \) towards the goal of selecting a firing instant that is closer to that of clock \( j \). In this way the phase \( \Phi_i(t) \) is adjusted. The convergence and the stability of this method can be studied using Lyapunov stability and is shown to depend on the property of the Laplacian matrix of the associated graph.

Despite the benefits derived from the pulse coupling, there are two large drawbacks:

1) it is hard to extend the analysis to realistic and complex scenarios with inaccurate clocks, propagation delays, or time-varying channels;
2) the system design is not flexible enough to grant degrees of freedom for the achievement of additional relevant goals (trading complexity for accuracy, security...)

\[ x_i(t) \]

\[ t_j(n) \]

\[ \varepsilon(x_i(t)) \]

\[ e \]

\[ \varepsilon(\cdot) \]

\[ \varepsilon, \mu \]

\[ \varepsilon_0 \]

\[ \frac{z^{-1}}{1-\mu z^{-1}} \]

\[ \frac{1-z^{-1}}{1-\mu z^{-1}} \]

\[ \Phi_i(t) \]

\[ x_i(t) \]

\[ t_j(n) \]

\[ u/T_{\text{nom}} \]

\[ u/T_{\text{nom}} \]

\[ 0 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 4 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]
Figure 3. The system of three pulse-coupled discrete-time PLLs

**IMPACT OF TOPOLOGY AND EFFECTS OF SHADOWING**

The convergence properties of distributed synchronization depend on the topology of the network of randomly located nodes with weights $\alpha_{ij}$. Assuming log-normal shadowing and considering the clock standard deviation $\xi(n)$ of a pulse-coupled PLLs, the power received over distance $d_{ij}$ is $P = 10^{-\nu/10} / d_{ij}^\beta$, where $\nu$ a zero-mean Gaussian random variable with standard deviation $\sigma$.

It can be seen that increasing the amount of shadowing in the model ($\sigma$) improves both the convergence speed and the phase error of the system of distributed PLLs. This is because distributed agreement on a graph improves if the graph has the features of a small-world network, where the paths are made of a small number of edges between any two nodes.

Shadowing breaks a few close connections and, due to the long tails of the log-normal distribution, creates a few long links, enhancing the small-world properties of the connectivity graph.

Figure 4. Small-world effects of shadowing: Standard deviation of the clocks for discrete-time PLLs vs time $n$ for different values of the standard deviation of shadowing $\sigma$.

**REFERENCES**
