Continuously – Coupled Clocks

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Introduction

Communication is a fundamental act taking place in our everyday lives. Humans are being trained from the very early stages of our lives to do so, but what has become of interest since the start of the semiconductor era, is to migrate the ability of communication to artificial agents. In other words, give the ability to an entity or a group to convey information to one another via the use of a mutually understandable language.

In order to do so, the various entities need to have some sort of a reference mechanism to coordinate the arbitrary tasks that they are responsible for – including communicating with one another. This work is a report of the distributed synchronization mechanism for decentralized wireless networks.

The document is organized as follows. Next, we are going to define synchronization and name some functionalities that is enables as well as some challenges that it faces. Following, we are going to introduce the continuously coupled analogue clocks on which two models base their functionality. Kuramoto’s model and the Continuously coupled Linear PLLs.

Synchronization in Wireless networks

Synchronization is essentially the mechanism that via the exchange of local time information allows various entities to achieve and maintain coordination. In traditional wireless networks i.e. cellular telephony, synchronization is achieved by exploiting a master-slave structure but due to a series of applications that have emerged the last years i.e. ad hoc sensor or vehicular networks, there has been a lot of research trying to investigate further and provide solutions to the necessities that have come to be. Most certainly, there are plenty of applications that could benefit from synchronization in a distributed fashion such as:

- Signal processing applications
- Spectral energy-efficient networking
- Cooperative transmission

But, creating such a mechanism can be proven challenging since there exist random delays between transmission and reception of a timing signal -due to propagation and processing latency on both sides, but also designing challenges since such networks have to account for:

- Energy Efficiency
- Scalability
- Application specificity

To continue with, we have to provide some information concerning the clocks and synchronization. To begin, we have the uncoupled clocks where all the nodes do not exchange any information between them, hence synchronization cannot be achieved. Then, we have the coupled clocks which can achieve frequency synchronization or full synchronization (i.e. phase and frequency). So following we are going
to be talking about the Continuously Coupled Analog Clocks on which the Kuramoto’s model and the Continuously Coupled Linear PLLs base their functionality.

**Continuously Coupled Analog Clocks**

An analog clock is basically characterized by an oscillator:

\[ s_i(t) = \cos \Phi_i(t) \]

Whose phase evolves as:

\[ \Phi_i(t) = \Phi_i(0) + \frac{2\pi}{T_i} t + \xi(t) \]

Each node transmits a signal proportional to its local oscillator \( s_i(t) \), and updates the instantaneous phase \( \Phi_i(t) \) based on the signal received from other nodes. The scheme assumes full duplex.

**Phase Detector**

\[ \Delta \Phi_i(t) = \sum_{j=1, j \neq i}^N \alpha_{ij} \cdot f(\Phi_j(t) - \Phi_i(t)) \]

**Local phase update**

\[ \dot{\Phi}_i(t) = \frac{2\pi}{T_i} + \varepsilon_0 \sum_{j=1, j \neq i}^N \alpha_{ij} \cdot f(\Phi_j(t) - \Phi_i(t)) \]

**Kuramoto’s Model**

This model is the first model of coupled analog oscillators, which was motivated from the behavior of chemical and biological oscillators. It is a sinusoidal phase detector, a first order PLL with \( \varepsilon(s) = \varepsilon_0 \) that assumes all-to-all connectivity and coupling amongst the clocks. A very famous experiment that can be modelled by Kuramoto’s model is the synchronization of the metronomes as it can be seen below.

![Figure 1 Metronomes Synchronization (source: wiki/Kuramoto_model)](image)
**Continuously Coupled Linear PLLs**

This model is a linear phase detector of the type \( f(x) = x \), with an arbitrary number of connections under the constraint of convexity and loop filters of the type \( \varepsilon(s) = \varepsilon_0 / (1 - s/\mu) \) (Second order PLL). Linearization, allows us to use algebraical tools in order to study the convergence of such systems that are modeled based on the following differential equation:

\[
\dot{\Phi}(t) = \omega - \varepsilon_0 \cdot L \Phi(t)
\]

Where:

- \( \Phi(t), \omega \) are vectors
- \( L \) is the graph Laplacian associated with the connectivity graph that describes the network

More specifically, the synchronization convergence of the system depends on the network topology. Guaranteed asymptotic stability exists when the graph is strongly connected – there exists at least one path between every pair of nodes.

**References**