Chimera states: An unexpected phenomenon in the synchronization of networks

Simultaneous occurrence of synchrony and chaos in coupled oscillatory systems

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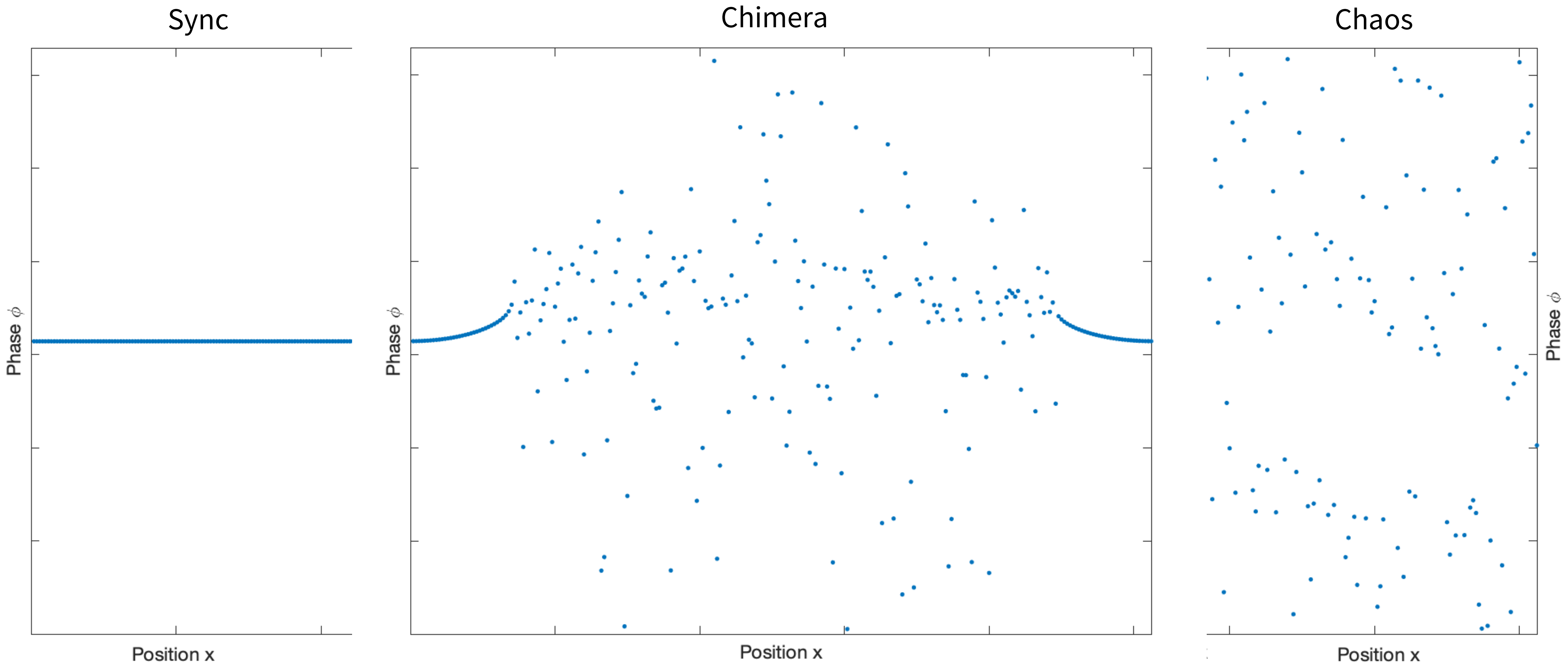

Reference: Own simulations based on [2]: Kuramoto & Battogtokh (2002)

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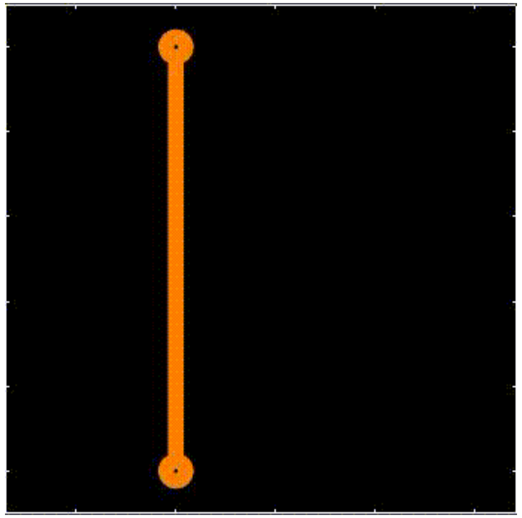
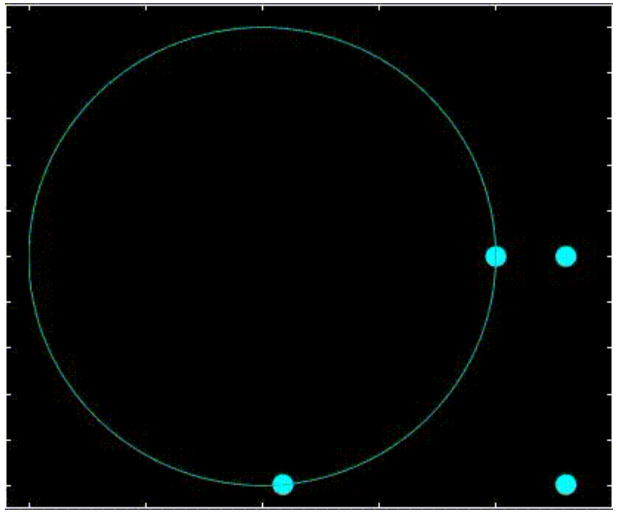
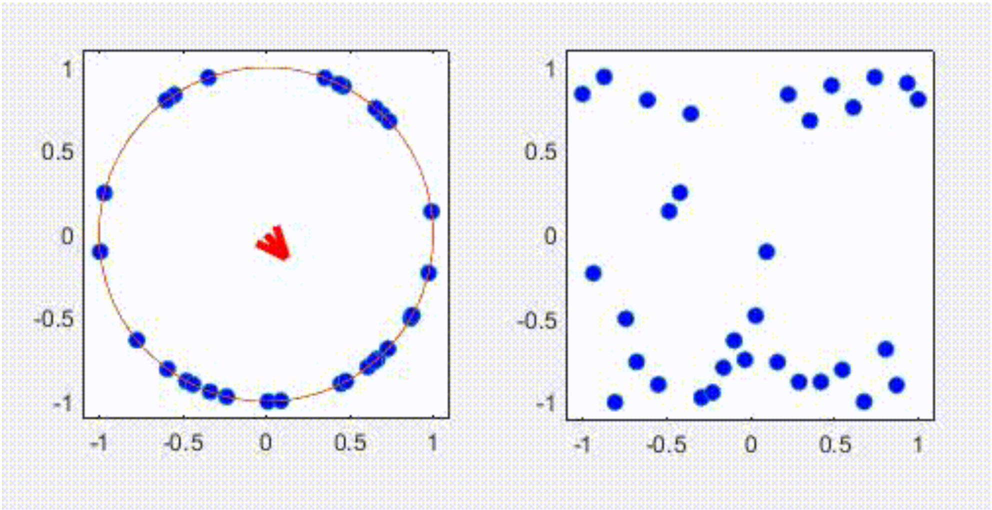
Coupled oscillatory systems basically consist of elements with repeating behaviour, influencing each other.



Example: Pendulum



Circular representation



Group of synchronizing oscillators

References: [3] Ortiz, Tavarez, Ruben

Kuramoto‘s model [1] was the first solvable model of coupled phase oscillators.

Globally coupled phase oscillators [1]:

$$\dot{\phi}_i = \omega_i - \sum_{j=1}^{N} F_{ij}(\phi_i - \phi_j), \quad i = 1, \ldots, N$$

$$F_{ij}(\phi_i - \phi_j) = \frac{k}{N} \sin(\phi_i - \phi_j)$$

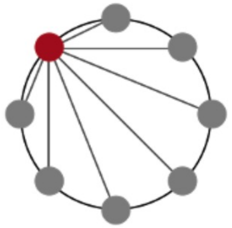


Figure:
Global coupling

ωi … natural frequencies (Gaussian distribution)
Fij … coupling function
k …. coupling constant
Ω … common frequency

Global states:

- **Chaos:**
  Phase differences are not constant in time, no common frequency, not predictable
- **Sync:**
  Phases evolve at the same speed
- **Phase-Locked:**
  Phases evolve at the same speed; Phase difference is is not zero but constant
- **Partially Synchronized:**
  Some oscillators fall into sync, some not; frequencies too different from the mean may not fall into sync

[1]: Kuramoto (1975)

Kuramoto & Battogtokh [2] unexpectedly discovered the chimera state during their work on an adapted model.

Model adaptations [2]:

- **Deterministic distribution on a circle**
- **Non-local coupling**
- **Equal initial frequencies**
- **Initial phase states drawn at random from a gaussian distribution**
- **Introduction of a phase gap**

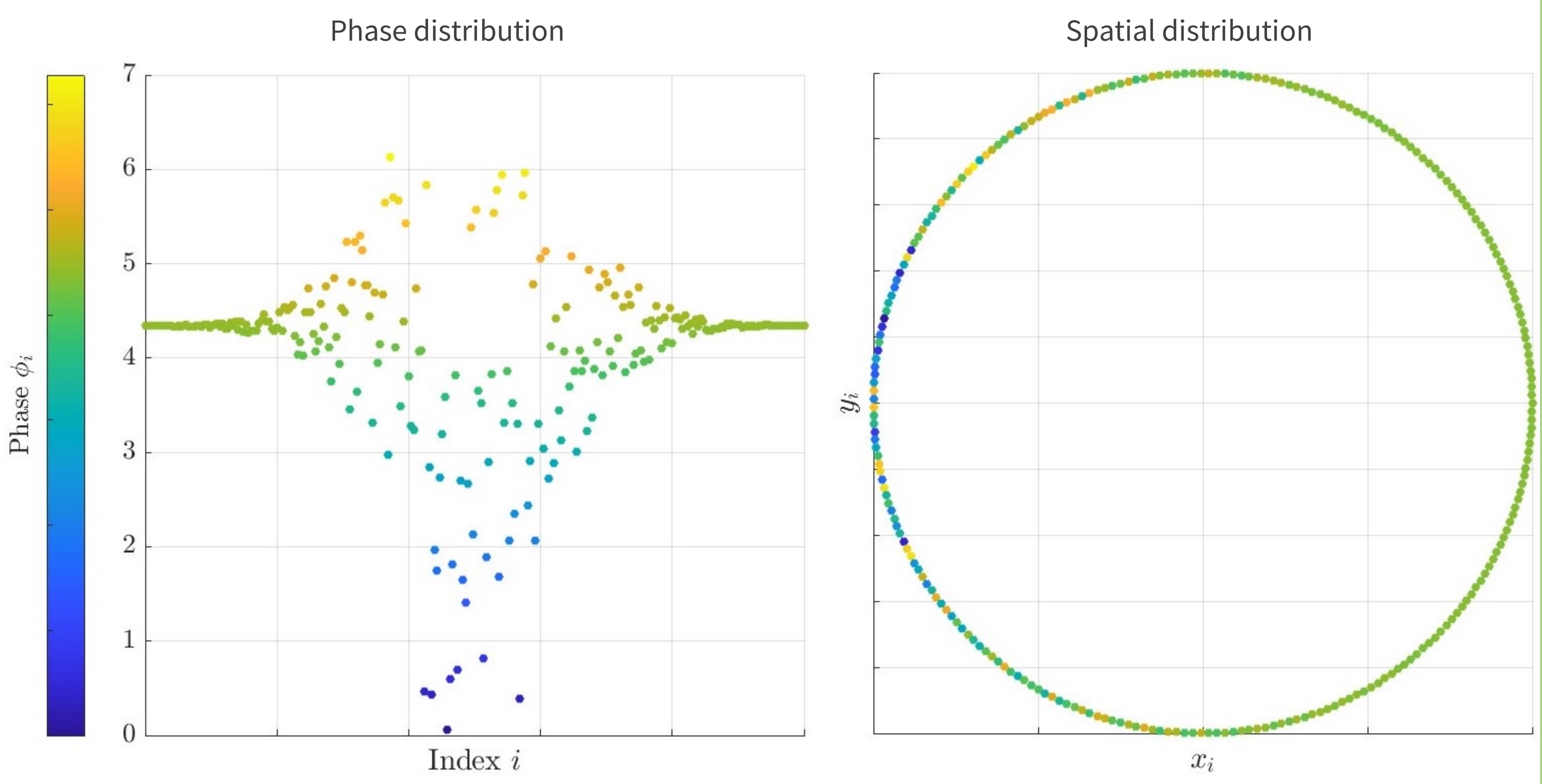


Figure:
Simulation of a chimera state based on [2]:
Phases of N=256 oscillators on a circle mapped to a colormap.

[2]: Kuramoto & Battogtokh (2002)

Kuramoto‘s model was the first solvable model of coupled phase oscillators.

Globally coupled *phase* oscillators [1]:

$$\dot{\phi}_i = \omega_i - \sum_{j=1}^{N} F_{ij}(\phi_i - \phi_j), \quad i = 1, \ldots, N$$

$$F_{ij}(\phi_i - \phi_j) = \frac{k}{N} \sin(\phi_i - \phi_j)$$

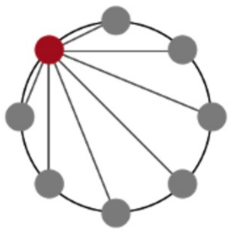


Figure:
Global coupling

$$\omega_i = random$$

ωi … natural frequencies (Gaussian distribution)
Fij … coupling function
k …. coupling constant
Ω … common frequency

[1]: Kuramoto (1975), [2]: Kuramoto & Battogtokh (2002)

Kuramoto‘s model adjusted to non-locally coupled identical oscillators distributed on a circle exhibited chimera states.

Globally coupled *phase* oscillators [1]:

$$\dot{\phi}_i = \omega_i - \sum_{j=1}^{N} F_{ij}(\phi_i - \phi_j), \quad i = 1, \dots, N$$

$$F_{ij}(\phi_i - \phi_j) = \frac{k}{N}\sin(\phi_i - \phi_j)$$

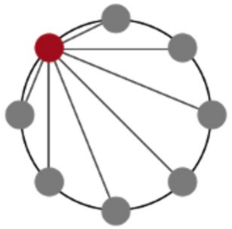


Figure:
Global coupling

$\omega_i = random$

ωi … natural frequencies (Gaussian distribution)
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Ω … common frequency

Non-locally coupled *phase* oscillators on a *circle* [2]:

$$\dot{\phi}_i = \Omega - \sum_{j=1}^{N} K_{ij}\sin(\phi_i - \phi_j + \alpha)$$

$$K_{ij} = Ce^{-\kappa \cdot d_{ij}}$$

Figure:
Non-local coupling

$\omega_i = \Omega$ = const.

Kij … coupling kernel
dij . .. distance function
α … phase lag

[1]: Kuramoto (1975), [2]: Kuramoto & Battogtokh (2002)

Kuramoto‘s chimera model is further developed to a pulse coupled model.

Non-locally coupled *phase* oscillators on a *circle* [2]:

$$\dot{\phi}_i = \Omega - \sum_{j=1}^{N} K_{ij} \sin(\phi_i - \phi_j + \alpha)$$

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Figure:
Non-local coupling

$\omega_i = \Omega$ = const.

Kij … coupling kernel
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α … phase lag

Non-locally coupled *pulse* oscillators:

Phase response function:
The ith oscillator's phase upon receiving a pulse from j

$$h(\phi_i)_j = (\phi_i - K_{ij} \sin(2\pi\phi_i + \alpha)) \bmod 1$$

Figure:
Non-local coupling

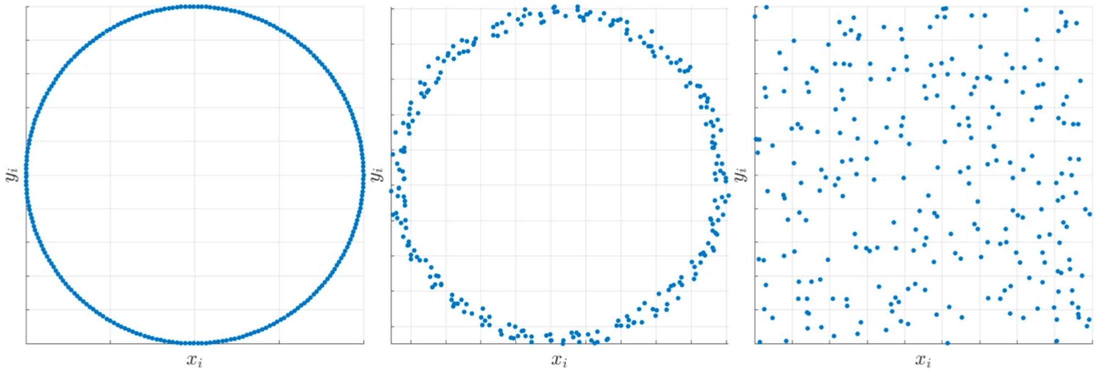


Figure:
Spatial distributions for which the model has been tested and has led to chimera states.

[2]: Kuramoto & Battogtokh (2002)

The phase response function gives some insights into the system's behavior.

$$h(\phi_i)_j = (\phi_i - K_{ij}\sin(2\pi\phi_i + \alpha)) \bmod 1$$

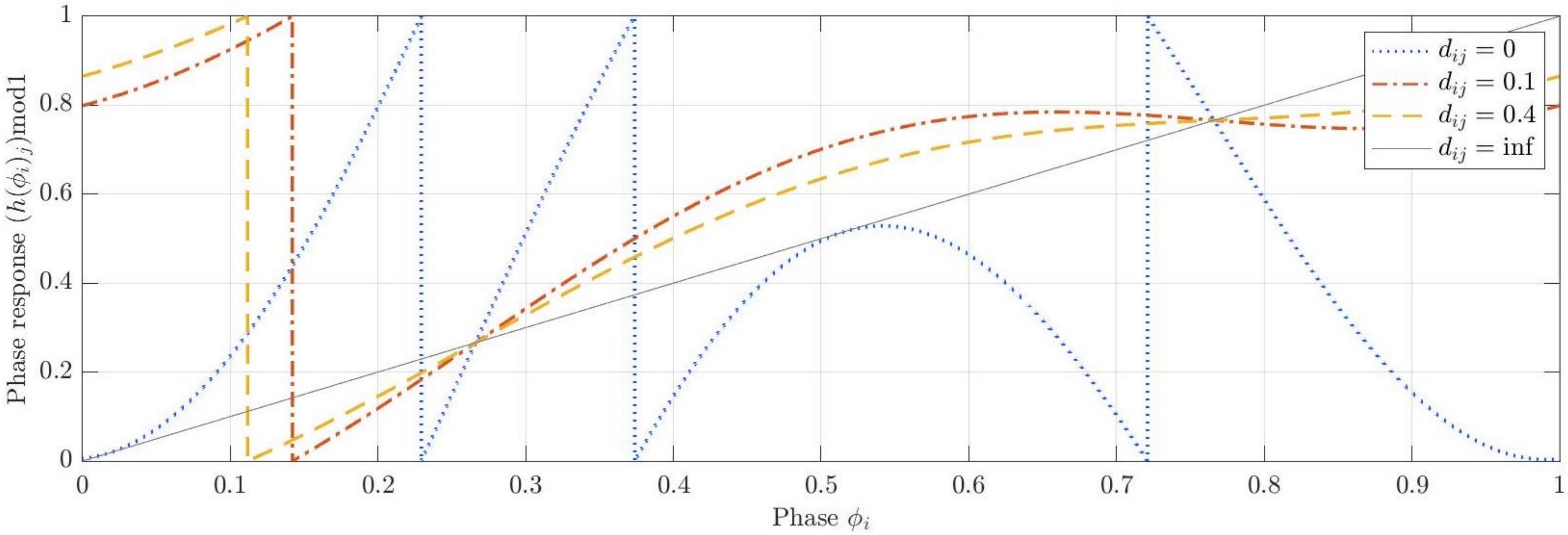
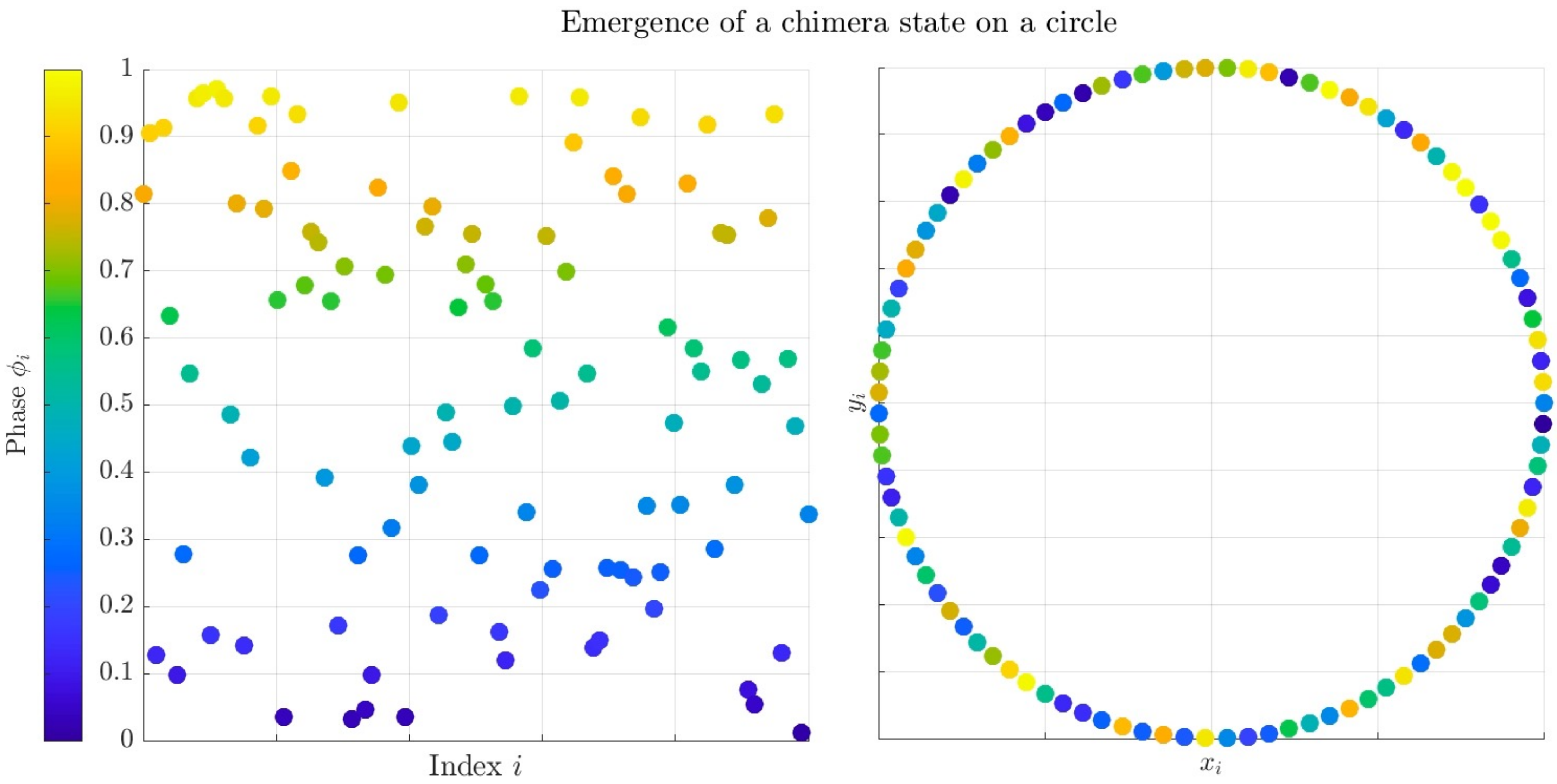


Figure:
Phase response functions for pairs of oscillators with different distances.

- Nodal points are fixed points of the dynamical system
- Synchronizing phases are drawn towards the stable fixed points at 0.78.

Video Demonstration



Can chimera states be useful? – Some abstract use cases

Wireless (sensor-)networks:

- **Interference & collision avoidance:**
  - distribute sending times based on phases of chaotic subset
  - exploit synchrony in other subset
- **Hidden terminal problem avoidance:**
  - exploit chaotic distribution of sending times
  - exploit synchrony of other subset
- **Reduce sampling burden:**
  - When sensing has to be performed in many points of time -> distribute chaotically
- **Sleep Mode Scheduling**

Production plants:

- **Job Shop Production Scheduling**
  - Using bottom-up optimization instead of linear optimization (similar to swarm intelligence applications in smart factories [4])
- **Bottleneck Avoidance:**
  - Similarly to collision avoidance
- **Maintanance Scheduling**
- **Coexistence of centralized control and self-organization:**
  - Chaotic and synchronous subsets underly different control mechanisms

Other analogies:

- **Partial supply chain disruption**
  - Complex supply chain models [5] may be further developed using chimera states
- **Partial synchronization of behavior on financial markets**
  - Synchrony can lead to stock market crashed [6]
- **Spatial population dynamics**
- **Organizations:**
  - Well structured established companies/departments vs. innovation hubs

[4]: Schranz, et al. (2021), [5]: Raaidi et al (2018) , [6]: Peron et al. (2011)

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Conclusio

- The emergence a chimera state (coexistence of synchrony and chaos) within a single network is possible
- Chimera states in pulse coupled oscillatory systems need to be further studied
- Applications for chimera states and the different models may be given but are not analyzed in more detail yet

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Q & A

References

[1]: Kuramoto, Y. (1975). Self-entrainment of a population of coupled non-linear oscillators. In International symposium on mathematical problems in theoretical physics (pp. 420-422). Springer, Berlin, Heidelberg.

[2]: Kuramoto, Y., & Battogtokh, D. (2002). Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. *arXiv preprint cond-mat/0210694*.

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[4]: M. Schranz, M. Umlauft, and W. Elmenreich. Bottom-up job shop scheduling with swarm intelligence in large production plants. 2021.

[5]: S. Raaidi, I. Bouhaddou, and A. Benghabrit. Is supply chain a complex system? MATEC Web of Conferences, EDP Sciences, 200:18, 2018.

[6]: T. Peron and F. Rodrigues. Collective behavior in financial markets. EPL (Europhysics Letters), 96(4):48004, 2011.